Heat and Mass Transfer in Spray Drying

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Studies of the evaporation and drying rates in an experimental concurrent spray dryer, 8 in. in diameter and 14 ft. high, are reported for various operating air temperatures. It was found that the total evaporation and drying time could be accurately predicted by employing a step-by-step method of calculation. Owing to the probable absence of internal diffusional resistance in the small particles, less than 30 μ, produced in the dryer, no significant falling-rate period was observed. This was in marked contrast with tray-drying experiments carried out on the same substance, under similar drying conditions. The results also confirmed that a Nusselt number of 2 can safely be used in spray-drying heat transfer calculations.

The evaporation and subsequent drying of the atomized droplets in a spray dryer involve simultaneous considerations of heat, mass, and momentum transfer. Similar considerations also form the basis of related operations of chemical engineering such as spray cooling, flash drying, spray crystallization, and cyclone evaporation. In spite of the growing industrial importance of these various operations there are still certain aspects of the mechanism of evaporation and drying of atomized sprays which have not been sufficiently investigated to permit the formulation of sound design criteria. In particular the rate of evaporation of small droplets suspended in a turbulent gas stream has never been properly ascertained, nor has the rate of drying of such droplets been experimentally determined under conditions likely to be encountered in spray dryers.

Most of the research pertaining to heat and mass transfer in spray drying has been carried out on single, relatively large, stationary drops of pure solvent. The more important contributions are due to Froessling (1) who made an experimental as well as a theoretical analysis of the problem, and to Ranz and Marshall (2) who in 1952 presented the following equations for heat and mass transfer to a drop under forced or natural convection:

$$N_{Nu} = 2 + 0.60 \ (N_{Re})^{1/2}(N_{Pr})^{1/3} \quad (1)$$
$$N_{Nu'} = 2 + 0.60 \ (N_{Re})^{1/2}(N_{Sc})^{1/3} \quad (2)$$
$$N_{Nu} = 2 + 0.60 \ (N_{Gr})^{1/4}(N_{Pr})^{1/3} \quad (3)$$
$$N_{Nu'} = 2 + 0.60 \ (N_{Gr})^{1/4}(N_{Sc})^{1/3} \quad (4)$$

The Nusselt number as given in these equations is termed the "apparent" Nusselt number and applies only at low evaporation rates. At higher rates the diffusing vapor leaving the drop surface changes the temperature gradient in the gas film surrounding the drop, and use of a correction has been recommended by several workers (3, 4, 5, 6).

The reliability of Equations (1) to (4) has been amply demonstrated, but to permit the calculation of the evap-

oration rate, the temperature of the droplet must of course be known. With pure solvents there is no problem, since the drops will be at or at least very close to the wet-bulb temperature of the air, provided that dynamic equilibrium is attained and that the surroundings are not at too high a temperature (2, 4). In the presence of a solute however the vapor pressure of the solvent will be depressed, and the droplet temperature will rise. From a rather limited study Ranz and Marshall (2) suggested that a saturated film is established very rapidly at the drop surface and that the rate of evaporation will correspond to saturated conditions, independent of the bulk concentration. Their work was done on single drops of ammonium nitrate solutions (about 1 mm. in diameter) suspended from a capillary, through which solvent was added continuously to maintain the diameter constant throughout the test, and the Reynolds number was 66.6. Although it is apparent that under these conditions a saturated film does form, later work at the University of Wisconsin indicated that this mechanism did not apply to certain soluble salts (7), and in any case it is doubtful whether it will prevail under actual spray-drying conditions.

The same general theoretical considerations apply for heat transfer to clouds of particles as do to single drops, but again certain complicating factors must be taken into account, such as the turbulence of the conveying gas stream, the particle-size distribution, and the effect of the lowered vapor pressure due to the presence of a solute (8).

Kessler (9) studied experimentally the diffusion of atomized sprays in turbulent air streams and concluded from his results that the eddy diffusivities of the particles and of the gas stream were essentially equal. Using alcohol drops he also found that the Nusselt number had a numerical value of approximately 2, although his study of the evaporation rate was only of a secondary nature. This would indicate then that the relative velocity between

the particles and the gas stream is negligible despite the turbulent eddies. Soo (10) made a theoretical analysis of this problem, and the equation he presented tends to confirm this conclusion, at least for small particle diameters.

Further evidence has also been presented by the authors (11) in a study of the heat and mass transfer coefficients to clouds of water droplets in turbulent air streams. For mean Sauter diameters ranging from 11.5 to 38.5 μ, the Nusselt number was found to have the value 2.07 ± 0.06 under various conditions of air velocity and temperature, which is in good agreement with Kessler's conclusions.

Marshall (12) also used a Nusselt number of 2 in proposing a step-by-step method for the calculation of the rate of evaporation of a cloud of water droplets of a given size distribution.

The question of a falling-rate period* in spray drying has received little attention, principally because of the difficulties associated with the experimental measurements. The calculation of particle temperature and the estimation of diffusional resistance in the final stages of drying still remain largely a matter of trial and error (8). Some qualitative information can be obtained from the examination of the physical state of the spray-dried product (13), but such observations are inadequate for quantitative calculations.

EXPERIMENTAL

In most conventional spray dryers the particle trajectories and gas-flow patterns are of such complexity that it is difficult to follow the progressive evaporation and drying of the particles. A special vertical concurrent spray dryer was therefore built in which all the pertinent variables could be measured along the drying path. The rate of evaporation and drying of Lignosol (trade name of dried calcium spent sulphite liquors, consisting essentially of calcium lignosulphonate) was determined in this chamber under various operating conditions, and the experimental results were compared with theoretical calculations. No attempt was made in this study to take measurements close to the nozzle, as this was the object of a separate investigation (14).

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* Although the rate of evaporation from a solution steadily falls as the solute concentration increases, the term "falling-rate period" as used here is restricted to its conventional meaning and applies only to the drying of the solid formed near the end of the operation.

TABLE 1. OPERATING DATA AND EXPERIMENTAL RESULTS
RUN NO. 20

Feed rate — 5.68 lb./hr. Feed concentration — 18.2% Feed temp. — 86°F.

Atom. air pressure — 20 lb./sq. in. gauge Drying air rate — 891 lb. dry air/hr.

x, ft.	t, °F.	H, lb. water / lb. dry air	D_{vs}, μ	X,	X_{hb},	X_{mb},
				lb. water /lb. Lignosol		
0	147.0	0.0031	—	4.49	4.49	4.49
1.46	134.0	—	—	—	2.00	
1.79	—	0.0064	28.9	1.86	—	1.65
1.96	129.9	—	—	—	1.22	
2.86	126.5	—	—	—	0.45	
3.19	—	0.0079	21.1	0.42	—	0.36
4.86	125.3	—	—	—	0.19	
5.19	—	0.0082	16.1	0.02	—	0.10
6.86	125.2	—	—	—	0.10	
7.19	—	0.0083	16.9	0.02	—	0.01
8.86	125.1	—	—	—	0.10	
9.19	—	0.0084	15.8	0.01	—	0.00
11.19	—	—	15.0	0.01		

Equipment

The experimental spray dryer, a detailed description of which has been given previously (11), consisted of the same basic components as an industrial installation.

The well-insulated drying chamber was 14 ft. high and 8 in. in diameter. Sampling ports were provided throughout the length of the unit, and electric heaters with automatic control were used to heat the air to the desired temperature. The pressure in the chamber was maintained at nearly atmospheric value by means of a push-pull blower arrangement.

An internal-mixing pneumatic nozzle, was used for atomization of the Lignosol solution, and a pressurized feed tank provided a steady flow to the nozzle.

Methods of Measurement

To follow the progressive evaporation and drying of the droplets special sampling techniques had to be used which would permit conditions along the entire length of the drying chamber to be determined. In addition methods for the measurement of air temperature, humidity, particle size, and solid concentration were developed.

Air temperature. The presence of liquid droplets in the air stream necessitated the use of shielding. Thermocouple probes were found to be unsatisfactory. A semicircular shield, 0.4 in. in diameter and 1.5 in. long, installed on the bulb of a precision mercury thermometer (0.2-in. diameter) was found to give the best results, particularly when the stem of the thermometer was protected with glass tubing to minimize heat losses. Calibration checks (11) have shown that drop impingement on the bulb was insignificant and that the true gas temperature was determined.

Air humidity. The more common methods of humidity determination were rejected either because they lacked sufficient accuracy or because they required too large an air sample. A special volumetric method of measurement was therefore developed.

In essence the technique consisted of absorbing the water vapor from a known volume of air by contact with magnesium perchlorate and of noting the resulting change in volume. A precision 100-ml. mine-air gas burette was used, calibrated in 0.05-ml. subdivisions in the upper range. Calibration tests carried out with air of known humidity showed that under rigidly constant conditions of temperature and pressure results with an error of less than 1% could be obtained (11).

Air samples were drawn from the drying chamber countercurrently to the main stream, through an inverted aluminum cup, 0.75-in. I.D., which was inserted into the chamber at a predetermined sampling port. The air was sucked into an evacuated sampling bottle at such a rate that the air velocity in the cup was well below the average terminal velocity of the droplets, thus achieving the desired gas-liquid separation.

Particle size. The method developed by J. H. Rupe (15) was used to determine the droplet size and size distribution along the chamber. It consisted of collecting a sample of the spray in a small immersion cell, 0.22-in. O.D. and 0.18-in. I.D., the bottom of which was made of optical glass and filled with Varsol. The glass was coated so that the droplet remained spherical. The sample was obtained by quickly traversing the spray with the cell held in a special holder. Because of the very low surface tension between Varsol and water no tendency for the drops to coalesce was observed. The target efficiency was excellent, as shown by the number of very small droplets which were collected. The droplets were photographed immediately upon sampling, and from 150 to 300 particles were measured in each sample. To test the accuracy of the method, determinations were carried out on several samples collected under identical operating conditions, and the agreement was found to be excellent. A lognormal distribution was found to be approximated by the drop-size data, and at an atomizing air pressure of 20 lb./sq. in. gauge the maximum droplet size was found to be around 100 μ.
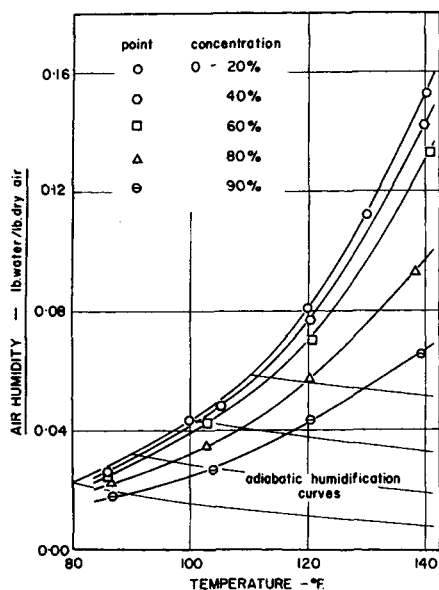

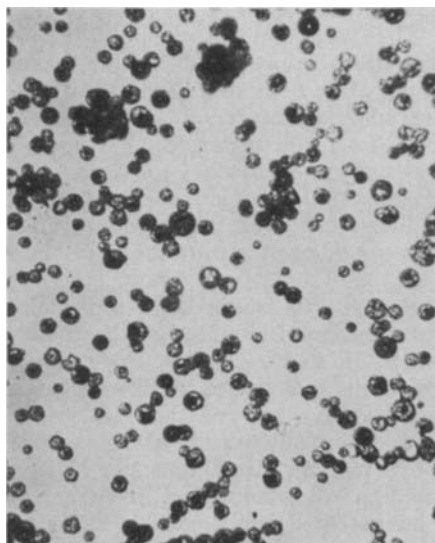Fig. 1. Saturation humidities of Lignosol solutions.

Fig. 2. Photomicrograph of Lignosol particles, run number 20 (220 X).
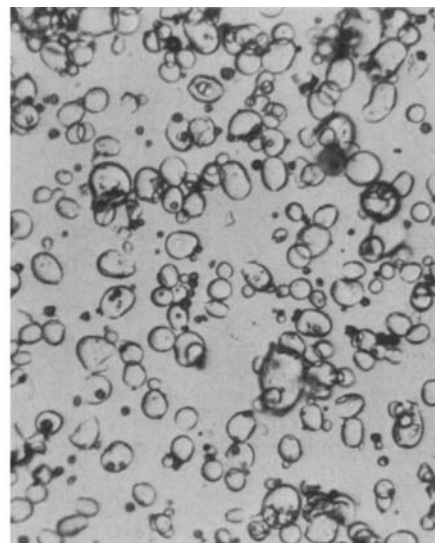

Fig. 3. Photomicrograph of Lignosol particles, run number 23 (220 X).

TABLE 2. OPERATING DATA AND EXPERIMENTAL RESULTS
RUN No. 23

Feed rate — 5.70 lb./hr.   Feed concentration — 18.2%   Feed temp. — 86°F.

Atom. air pressure — 20 lb./sq. in. gauge   Drying air rate — 530 lb. dry air/hr.

x, ft.	t, °F.	H, lb. water / lb. dry air	X_{hb} lb. water/lb. Lignosol	X_{mb}
0	419.0	0.0261	4.50	4.50
0.79	406.0	—	3.10	
1.12	385.5	—	0.85	
1.46	379.5	—	0.19	
1.79	—	0.0347	—	0.00
1.96	377.8	—	0.00	
2.86	377.8	—	0.00	
3.19	—	0.0348	—	0.00

Lignosol concentration. The concentration of Lignosol in the particles at various distances from the nozzle was determined on small samples of the spray by means of a colorimetric method.

The major problem was in collecting the sample, since it was essential that evaporation be stopped as soon as the droplets entered the sampler, while at the same time it was necessary to know the total mass of sample collected. A small immersion cell was therefore used, filled with a silicone fluid, immiscible with both water and Lignosol solutions. Since the vapor pressure of the former is only about 0.5 mm. Hg at 70° to 100°C., no measurable evaporation could occur and the net increase in mass of the tared cell corresponded to the mass of sample collected, which ranged from 25 to 150 mg.

Lignosol is brown in color when in the presence of water, and concentrations around 0.1% can readily be measured in a colorimeter with a 425 μ filter. The instrument was calibrated by means of standard solutions carefully prepared from Lignosol of almost zero moisture content. Following dilution of the samples from the cell the total Lignosol content, and hence the solids concentration of the spray, could easily be determined by this method.

Physical Properties of Lignosol

To analyze the operation of the spray dryer certain pertinent physical properties of Lignosol solutions were required. The density and specific heat capacity were readily determined by the use of standard laboratory methods, and the following approximate equations were obtained in the temperature range from 70° to 120°F.:

$$\rho_s = 62 + 25/(1 + X) \qquad (5)$$

$$C_{ps} = 1 - 0.61/(1 + X) \qquad (6)$$

The heat of solution was found to be negligible, but the vapor pressure of Lignosol solutions of various concentrations had to be determined over a range of temperatures. A dew-point apparatus immersed in a constant-temperature bath (16) was used for this purpose, and the results obtained are shown in the form of a humidity chart in Figure 1. It can be seen that up to a concentration of about 20% Lignosol solutions exhibit typical colloidal properties, while above that concentration a decrease in vapor pressure takes place.

Procedure and Results

A rigidly standardized procedure was used to make runs on the experimental spray dryer (11). The major precautions were concerned with the determination of heat losses to permit proper heat balances and the establishment of equilibrium conditions prior to the taking of any readings.

Four runs were made with Lignosol solutions used as feed under approximately the same conditions of air velocity, feed rate, feed concentration, and atomizing pressure, but with inlet drying-air temperatures ranging from 106° to 419°F. The operating conditions used and experimental results obtained for two temperatures are shown in Tables 1 and 2. In the first table

(run 20, inlet air temperature 147°F.) X denotes the solid concentration as determined experimentally. No such data could be obtained in run 23, shown in Table 2 (inlet air temperature 419°F.), because the fluid decomposed at the temperature used, and no samples could be collected for the determination of solid concentration. Samples were still taken for particle-size measurement however.

The values of X_{hb} and X_{mb} shown in the tables were calculated from heat and material balances respectively; they provided a valuable check on the accuracy of the data. Figure 2 shows a typical photomicrograph of the product obtained in run 20, while Figure 3 shows the hollow particles produced in run 23. Photomicrographs of samples taken at increasing distances from the nozzle showed clearly that the droplets were still liquid at an average concentration of at least 84% and that expansion of the particles in run 23 did not occur until the very last stages of drying.

To permit a comparison between spray drying and tray drying an experimental run was made in a standard laboratory tray dryer, by means of a transite tray with a dummy front and a 5- by 5- by ⅛-in. depression in the center, in which the Lignosol solution was placed. The air temperature, air humidity, and initial concentration of the solution for the tray-drying test were practically identical with the conditions used in run 20, and the drying curve obtained is shown in Figure 4. The drying rate was constant up to 65 min., at which time a thin crust started to form on the surface. The crust was completed after about 140 min. and assumed a hard consistency after 190 min., at which point all drying practically stopped in spite of the fact that the bulk-moisture content was only about 50%.

Discussion of Results

The good agreement between the observed and the calculated values of the particle moisture content $(X, X_{hb}, $ and $X_{mb})$, as shown in Figure 5, reflects the accuracy of the results. It should be noted that because the Lignosol concentrations were experimentally measured as a function of the distance from the nozzle, they there-
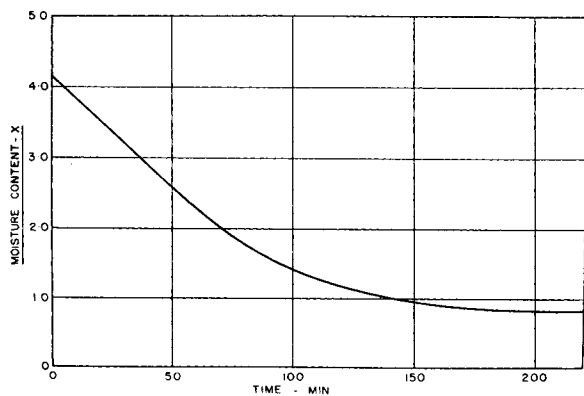

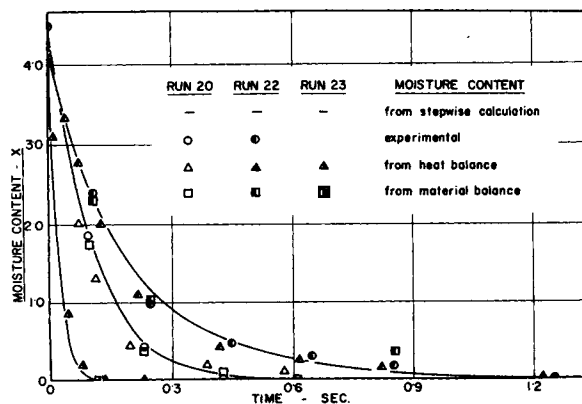
Fig. 4. Drying curve, tray-drying experiment.

Fig. 5. Drying curve, spray-drying experiments.

fore had to be expressed on the basis of time elapsed before comparison with theoretical calculations could be established. The conversion could readily be made for the so-called "fall zone" of the drying chamber, in which the particle velocity is the same as that of the drying air, since their terminal velocity is negligibly small. In the nozzle zone however the particle velocity is not known and extrapolation is necessary. When distance vs. concentration was plotted, a curve with an inflection at a distance of about 10-in. from the nozzle was obtained for all four runs. The lower portion of this curve, which applies to the fall zone, was therefore extrapolated, and the intercept between this curve and the feed concentration was taken as the zero distance from the nozzle for the purpose of time calculations.

Good agreement was also found between the experimental and calculated values of the air humidities, as shown in Figure 6 (run no. 22, inlet air temperature 106°F.) on which the observed variations in air temperature down the chamber are also indicated. The same type of extrapolation as explained above was again carried out, as indicated by the dotted lines.

Further evidence of the accuracy of the experimental data was obtained from the particle-size measurements. The mean volume diameter of the spray must be a function of concentration and density only, if the particles are not hollow and no agglomeration occurs, as shown by

$$(n\pi D_v{}^3)/(6) = (Z)/(c) \qquad (7)$$

Since the value of n was not known accurately, the experimental value of D_v at a distance of 1.79 ft. from the nozzle was taken as a basis, and the value of D_v was calculated from Equation (7) at various points (and hence various concentrations) down the chamber. Table 3 shows that excellent agreement with the experimentally observed values was obtained. This, incidentally, confirms the validity of the particle-size determinations, since in contrast to the estimation of D_{vs} the small particles are no longer unimportant in the measurement of D_v.
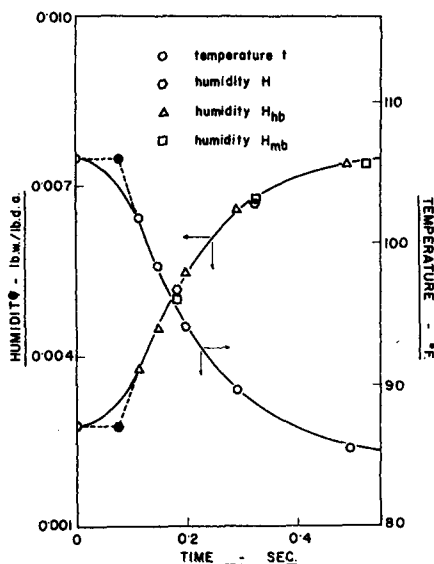

Fig. 6. Temperature and humidity distribution, run number 22.

Calculation of Evaporation and Drying Times

The experimental runs were carried out primarily to determine the observed evaporation and drying rates of Lignosol sprays under known conditions and to compare them with values calculated from theoretical considerations.

The theoretical approaches developed by Probert (17), Daskin (18), Miesse (19), and Shapiro and Erickson (20), among others, can be used for the estimation of the evaporation time only. When solid particles are formed, a step-by-step method of calculation must be resorted to, as advocated by Marshall (12), since it is no longer permissible to use average values for the physical properties of the particles. This is clearly indicated by Figure 7, which gives the size distribution (on a number basis) at various distances from the nozzle for run 20; it is obvious that the smaller particles are completely dried at a point where the larger ones are still almost at the feed concentration.

The calculations were therefore carried out by division of the particles into several groups, each of a given size range. For each group the rate of heat
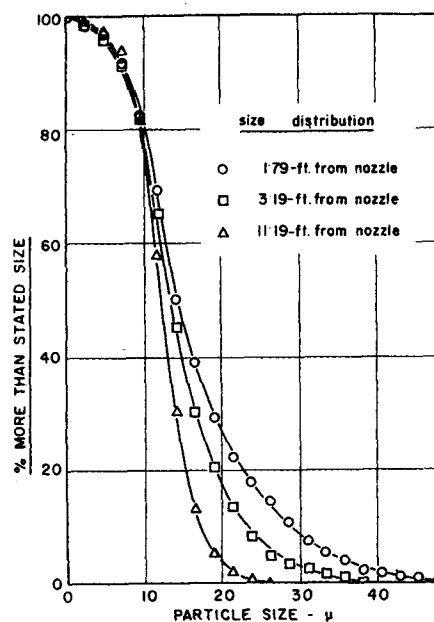

Fig. 7. Particle size distribution, run number 20.

transfer (when one neglects the sensible heat effects due to the increase in wet-bulb temperature) can be expressed as

$$(dQ/d\theta) = n_i h \pi D_i{}^2 (t - t_w) =$$
$$\lambda_w (dm/d\theta) \qquad (8)$$

It has been pointed out that the Nusselt number in Equations (1) and (3), and hence the heat transfer coefficient which it includes, is the apparent one. Godsave (6) has proposed the following equation for the calculation of the actual Nusselt number at the drop surface:

$$(N_{Nu})_{\text{actual}} = (N_{Nu})_{E=0}$$
$$[E(1/R_1 - 1/R_2)]/[e^{E(1/R_1 - 1/R_2)} - 1] \qquad (9)$$

In a discussion of this equation Marshall (3) points out that the actual value of the Nusselt number can be significantly lower than the apparent one at high evaporation rates, that is when E is large, and that the convective heat transfer coefficient for high air temperatures should be calculated from the equation

$$h_{\text{act}} = (h_{\text{app}})(a)/(e^a - 1) \qquad (10)$$

where $a = E(1/R_1 - 1/R_2)$. At low Reynolds numbers a is also equal to ln $(1 + \Delta t c_p/\lambda_w)$. Under the experimental conditions used in this study the correction indicated by Equation (10) was found to be negligibly small, and h_{app} was used in the calculations.

When one combines Equations (8), (10), and (7), the following expression is derived for each group:

$$dm = \frac{6h_{\text{app}} Z_i (t - t_w)}{D_i \rho_{si} c_i \lambda_w} d\theta \qquad (11)$$

TABLE 3. MEAN VOLUME DIAMETERS

x, ft.	RUN 20 Exp.	RUN 20 Calc.	RUN 21 Exp.	RUN 21 Calc.	RUN 22 Exp.	RUN 22 Calc.
1.79	21.6	21.6	20.3	20.3	22.2	22.2
3.19	17.7	16.9	16.6	15.7	—	—
5.19	14.9	15.1	15.2	14.7	15.8	16.0
7.19	14.4	14.6	14.3	14.2	—	—
9.19	14.3	14.5	—	—	—	—
11.19	13.9	14.5	13.7	13.9	—	—
13.19	—	—	—	—	13.9	13.9

TABLE 4. CALCULATED DRYING TIME FOR LIGNOSOL SOLUTIONS
(PARTICLE CONCENTRATION PERCENTAGE)

RUN 20

Diameter, μ				Z_i, lb. dry Lignosol/hr.				
2.4				0.0002				
7.2				0.0282				
11.9				0.3458				
16.7				0.4690				
21.4				0.1530				
26.2				0.0387				

θ, sec.	Dry particle diameter, μ						X_{avg}, lb. water lb. Lignosol	t, °F.
	2.4	7.2	11.9	16.7	21.4	26.2		
0	18.2	18.2	18.2	18.2	18.2	18.2	4.49	147.0
0.10	100	100	56.9	30.9	24.9	22.3	1.85	134.4
0.20	100	100	100	60.4	34.5	27.3	0.68	128.5
0.30	100	100	100	100	49.7	33.7	0.22	126.2
0.40	100	100	100	100	76.7	42.5	0.09	125.6
0.55	100	100	100	100	100	63.1	0.02	125.3

In this equation the particle concentration, density, diameter, and temperature for each group will be functions of dm, as will h_{app} [which can be obtained from $(N_{Nu})_{app} = 2$].

To carry out the calculations a knowledge of Z_i in each group is required. This is the only information which must be obtained experimentally, since it depends on the size distribution peculiar to the atomizing nozzle. In the present study it was obtained as follows: the spectrum of dry particles of Lignosol collected at the bottom of the chamber was divided into six groups of average diameters 2.4, 7.2, 11.9, 16.7, 21.4, and 26.2 μ, respectively. From the size-distribution curve the number of particles in each group, and Z_i for the group, could be calculated, if one remembered that $\Sigma Z_i =$ the rate of solids fed in the Lignosol solution per hour. Equation (11) was then used to calculate the change in concentration for each group in incremental intervals. More specifically the calculations were based on the following procedure:

1. For each size group the initial diameter of the droplet was back calculated from Equation (7) and its temperature obtained from Figure 1 with the bulk concentration of the feed used to determine the saturation vapor pressure.
2. The amount of water evaporated in the time increment was calculated from Equation (11) with the inlet conditions used for all the values in the equation and $(N_{Nu})_{app} = 2$ assumed.
3. The new conditions of air properties (temperature and humidity) and particle properties following evaporation occurring during the incremental period were calculated from the change in concentration that was obtained in

step 2.
4. The calculations in step 2 were repeated, but this time the prevailing conditions as obtained in step 3 were used.
5. The values of Δm obtained in steps 2 and 4 were averaged to give a more accurate change in concentration for each size group in the time interval.
6. The step-by-step calculations given in 1 to 5 were repeated for new time intervals, until the average concentration of the product was almost 100%. Corrections for the residual, equilibrium, moisture content of each fraction of particles were attempted but found to have negligible effect on the results.

The results of these step-by-step calculations for run 20 are summarized in Table 4, which shows the average values of the moisture content and the calculated values of the corresponding air temperature at 0.05-sec. intervals. Similar calculations were also made for runs 22 and 23, and the calculated moisture contents are represented by the continuous curves shown in Figure 5. The good agreement with the experimental points is obvious.

It should be pointed out that the particle-size distribution could not be determined experimentally from the dried product obtained in run 23, since the latter consisted of hollow particles which expanded during the last stages of the driving process. However since identical atomizing conditions and feed concentration were used in run 20, the distribution data obtained in the latter case were used instead.

CONCLUSIONS

The results of this study indicate that the drying time for a spray can be calculated with good accuracy, at least

for highly soluble materials such as Lignosol. A Nusselt number of 2 can be safely assumed, and the estimation of particle temperature should be based on the bulk concentration of the droplets. To carry out the calculations knowledge of the droplet-size distribution for the atomizing nozzle employed must be available. This may be obtained from a spray sample taken in the nozzle range if unheated air near saturation is admitted to the chamber to minimize evaporation, which is otherwise appreciable in that region (14), or it may be obtained from a size-distribution determination on the dry product, as was done in the present study. Care must be taken in the latter case that the particles are completely solid. Marshall pointed out that it is not a simple matter to produce solid particles by spray drying (12). Table 2 clearly shows however that solid particles of Lignosol were produced at low air temperatures. At higher temperatures (run 23) hollow particles were obtained in accordance with the general behavior (12, 13).

It is interesting to note that no significant falling-rate period (as defined earlier) could be detected, which is in marked contrast with the tray-drying experiment (Figure 4). In the latter the start of a very definite falling-rate period was detected at a moisture content of about 2.35 lb. water/lb. Lignosol, and drying almost stopped at a value of 0.83 lb. water/lb. Lignosol. In contrast Figure 5 and Table 4 indicate that the rate of moisture removal in the spray dryer is governed by the rate of evaporation under free liquid condition up to very high solid concentrations.

The difference lies in the concentration at which crust formation takes place. In the tray-drying experiment the depth of the sample was relatively large, and a heavy crust formed at a low bulk concentration. The diffusional resistance through this crust effectively reduced the evaporation rate. In the spray-drying experiments, on the other hand, the droplets were very small, and a crust did not form until the very last stages of drying; visual observations showed no evidence of crust formation below a solid content of 0.85.

Most authors argue from indirect evidence that the formation of a solid phase at the droplet surface introduces a diffusional resistance which slows down the rate of evaporation. The fact that residual moisture is almost always found in spray-dried products has been attributed to this cause (21) (in addition, of course, to equilibrium moisture considerations), although Table 4 shows that it may also be due to the comparatively long time required by

the larger particles to dry under free liquid condition. With solutions other than Lignosol it is possible that the falling-rate period will become significant, but this will have to be determined experimentally for each individual case.

ACKNOWLEDGMENT

NOTATION

a $= E(1/R_1 - 1/R_2)$;
c = solution concentration, lb. solids/lb. solution
c_i = solution concentration in ith group, lb. solids/lb. solution
C_{ps} = heat capacity of particles, B.t.u./(lb.)($°$F.)
C_p = heat capacity of water vapor, B.t.u./(lb.)($°$F.)
D = particle diameter, ft.
D_i = average particle diameter in ith group, ft.
D_v = mean volume diameter, ft.
D_{vs} = mean Sauter diameter, μ
E $= (dm/d\theta)(C_p/4\pi k_f)$
h = heat transfer coefficient, B.t.u./(hr.)(sq.ft.)($°$F.)
H = air humidity, lb. water/lb. dry air
H_{hb} = air humidity calculated from heat balance, lb. water/lb. dry air
H_{mb} = air humidity calculated from material balance, lb. water/lb. dry air
k_f = thermal conductivity of gas film surrounding the drop, B.t.u./(hr.)(ft.)($°$F.)

m = evaporation, lb.
n = rate of particles, No./hr.
Q = heat transferred to droplets, B.t.u.
R_1 = radius of particle, ft.
R_2 = radius of outer limit of gas film surrounding the drop, ft.
t = temperature of air, $°$F.
t_w = temperature of particles, $°$F.
X = moisture content, lb. water/lb. dry Lignosol
X_{hb} = moisture content calculated from heat balance, lb. water/lb. dry Lignosol
X_{mb} = moisture content calculated from material balance, lb. water/lb. dry Lignosol
x = distance from nozzle, ft.
Z = total rate of dry Lignosol, lb./hr.
Z_i = rate of dry Lignosol in particles in ith group, lb./hr.
Δm = amount of water evaporated
$\Delta \theta$ = time increment
θ = time, hr.
λ_w = latent heat of evaporation at the particle temperature, B.t.u./lb.
ρ_s = density of particles, lb./cu.ft.
ρ_{si} = average density of particles in ith group, lb./cu.ft.
N_{Gr} = Grashof number
N_{Nu} = Nusselt number
N_{Nu}' = modified Nusselt number
N_{Pr} = Prandtl number
N_{Re} = Reynolds number
N_{Sc} = Schmidt number

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The Effect of Dynamic Surface Tension on Nucleate Boiling Coefficients

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A study was made of the effects the surface tension of aqueous solutions of surface-active agents had on the bubble-formation characteristics of these solutions. Bubbles were formed by two processes: by passing air through a horizontal orifice submerged in the solution and by nucleate boiling at an electrically heated surface. The experimental measurements taken with the air-bubble system were used to determine the nature of the surface-tension effect on air-bubble formation. This knowledge was then applied to the interpretation of boiling measurements made with the same solutions. The boiling coefficients of the Tergitol-water solution were found to vary inversely with dynamic surface tension. Boiling coefficients for Aerosol-water solutions, however, varied in a manner which could not be related to surface-tension effects.

Numerous investigators have studied the role of surface tension in boiling

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and have reported widely contradictory results. Westwater (10) summarizes the results in terms of the value assigned to the exponent n in an assumed exponential relationship, $h \alpha 1/\sigma^n$; the range of values he reports is 2.5 to -1.275. As Westwater points out, there